Further analysis

Objective

Based on the understanding of the LV model from last report, further study the behaviours of the LV equations parameters.

Changes have been made

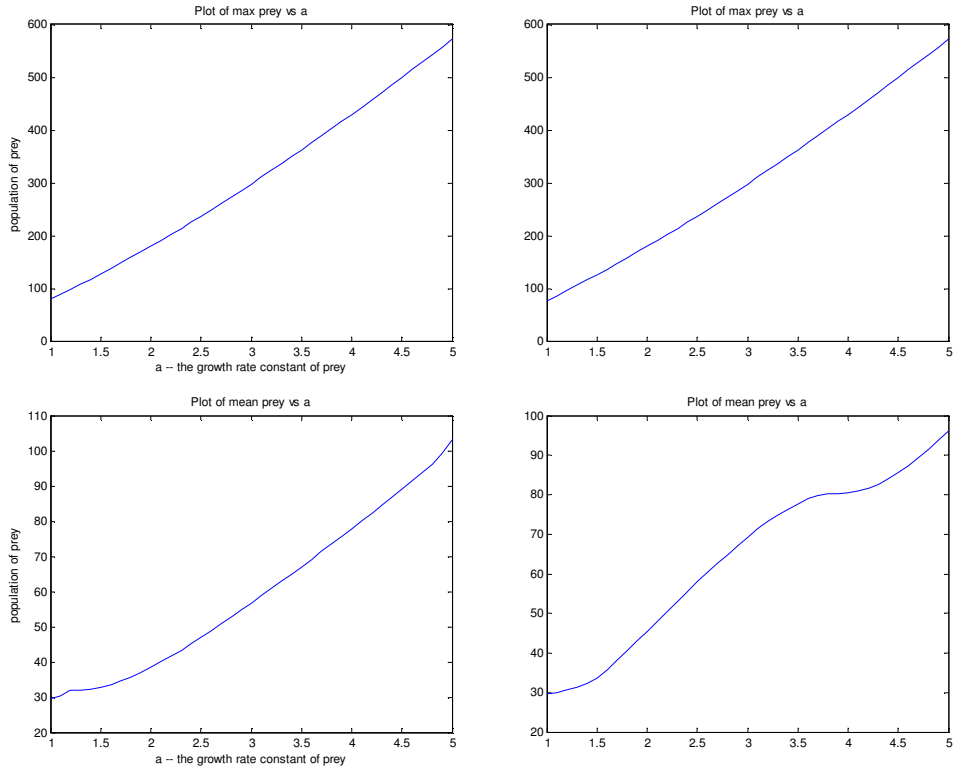
- ✓ Method of finding the mean is improved – instead of finding the mean of all the points, we are finding the mean of all the points within a period, and taking the average of this mean over several period. In this way, the points within the incomplete period would not contribute a shift to the actual mean.
- ✓ New factor included – the gap between the maximum peak and minimum peak within a set of points is found. In this way, we could have a rough approximation on the damping effect on the system

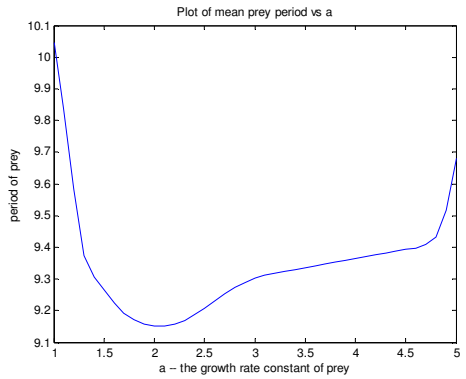
Methods

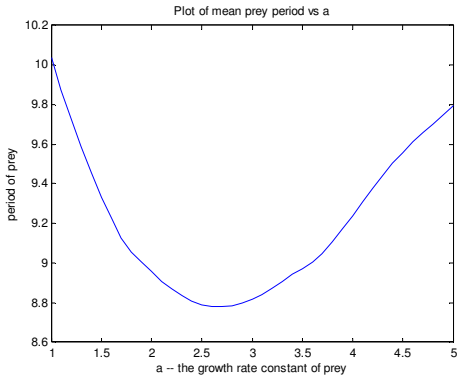
In this analysis, we are going to study the effect on increasing the time spans, so we have more points or cycles to analysis. We are also going to increase the range of parameter to 10 times of the original value instead of 5 times to verify our prediction in our previous report.

Experiment 4

In this experiment, we have increased our time spans to 100 instead of 20. Originally we only have 2 cycles, but now should be 5. Same condition as in Experiment 1 was applied





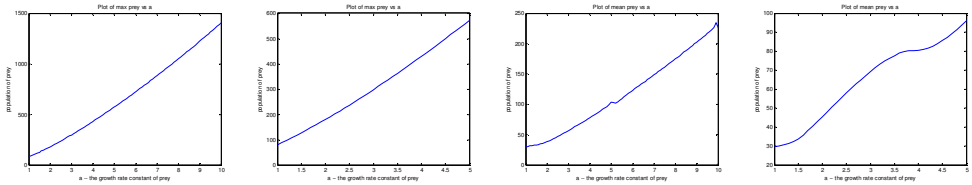


These graphs are analysis of the parameter a, the three graphs on the left were done with 100 time spans while the right were done with 20 times. We could observe that the graph on the left was more details than the right, but the general shapes are the same, hence we decided to use 100 time spans for the rest of the our experiments,

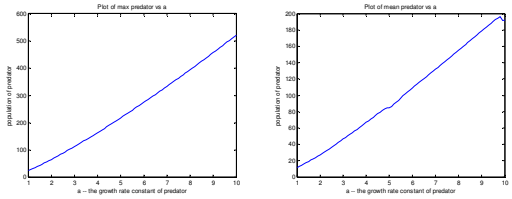
Experiment 5

The following graphs contains increased all the parameters by ten times with 100 steps, and the amplitude, mean, period, and damping amplitude of prey and predator was plotted against the varying parameters. Hence, there are 8 graphs for each varying parameter. (See appendix for full size graphs)

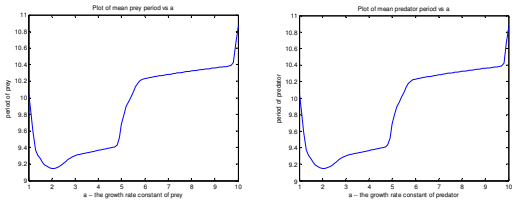
Analysis of parameter "a"



- From the left to the right, the first and the third graph are the maximum and the mean value of the prey from Experiment 5, and the one next to it are graphs from Experiment 1.
- It could be seen that the general trend does not change, and this further conclude our prediction.
- If we assume the change is linear, then the gradient for the maximum value graph is about 140, while the mean is 20.

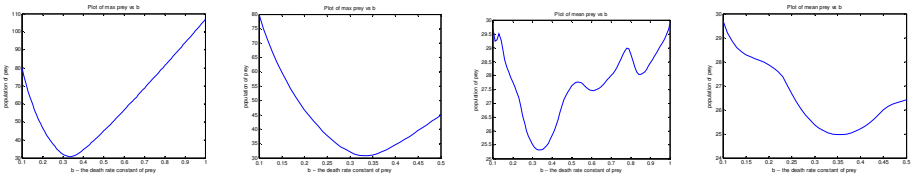


- Graphs for max and mean value of predator from Experiment 5
- The general population of predators is also increasing due to the abundance of prey

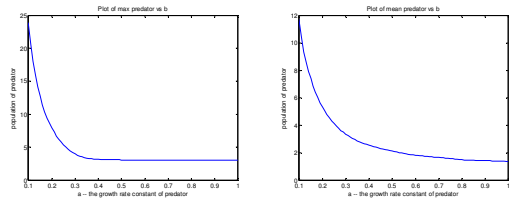


- Period graph for the prey and predator from Experiment 5
- The general trend is identical, the change in period is 10% from the mean

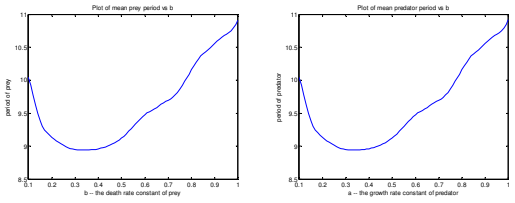
Analysis of parameter "b"



- From the left to the right, the first and the third graph are the maximum and the mean value of the prey from Experiment 5, and the one next to it are graphs from Experiment 1.
- It could be seen that the general trend does not change, and this further conclude our prediction.

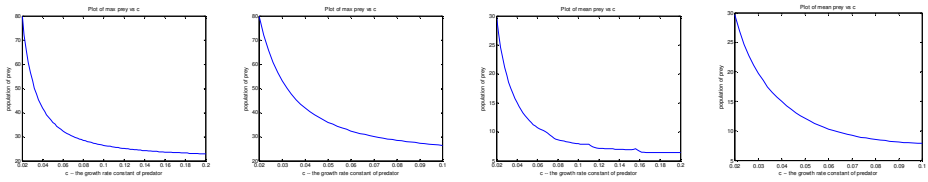


- Graphs for max and mean value of predator from Experiment 5
- As predicted from Experiment 1, the population of predator indeed reach its minimum value around b=3.0 to b=3.5. Hence verify our prediction on why prey has growth again when b>3



- Period graph for the prey and predator from Experiment 5
- The general trend is identical, the change in period within 10% from the mean

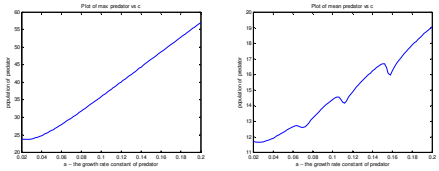
Analysis of parameter "c"



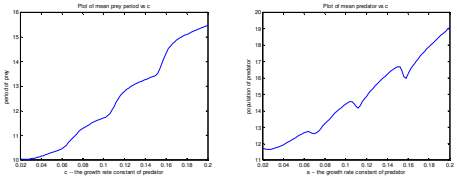
- From the left to the right, the first and the third graph are the maximum and the mean value

of the predator from Experiment 5, and the one next to it are graphs from Experiment 1.

- It could be seen that the general trend does not change, and we still cannot see the increase of prey at certain points. This might be due to the fact that the value of "c" in our experiment is too small compared to other parameters.

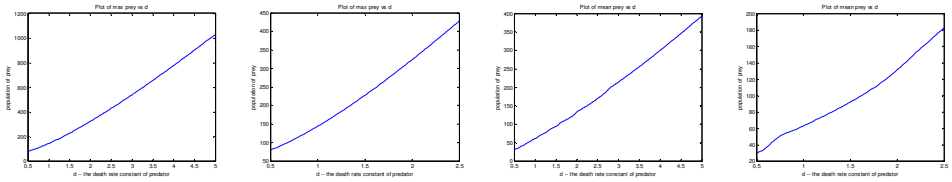


- Graphs for max and mean value of predator from Experiment 5
- We could see that the population of predator is increasing thus this is why the population is decreasing all over the range.

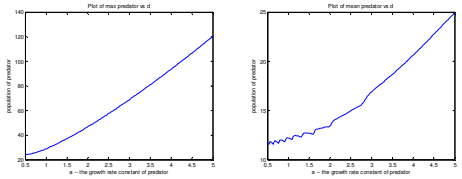


- Period graph for the prey and predator from Experiment 5
- The general trend is identical, there is an increased of period by 55%

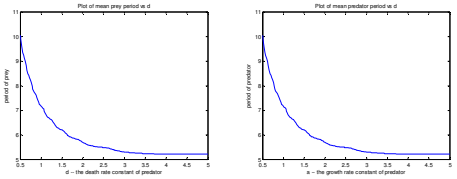
Analysis of parameter "d"



- From the left to the right, the first and the third graph are the maximum and the mean value of the predator from Experiment 5, and the one next to it are graphs from Experiment 1.
- It could be seen that the general trend does not change, and thus conclude our prediction.

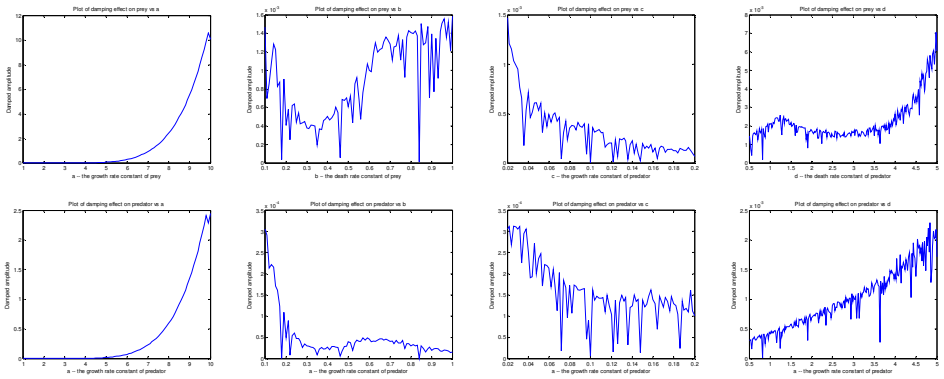


- Graphs for max and mean value of predator from Experiment 5
- It is interesting that the population of predator is increasing; this is probably due to the growth rate raised by the increased in prey population overwrite the death rate.



- Period graph for the prey and predator from Experiment 5
- The general trend is identical, there is a decreased of period by 50%

Analysis of the damping effect



- The above 8 graphs shows the damping effect of the system by increasing the parameters of "a", "b", "c", "d" respectively. The set above is for prey while the set below is for predator.
- However, the trend of the varying damping strength is not worth study, since this system should not have much oscillation. In fact, the absolute value changed is insignificant to the overall change of the populations – much less than 1%.
- The way to analysis our parameter might need to be modified by using "gap / maximum amplitude" instead.

Comment on future experiment

- After modification on the calculation method for the mean value of the population, we could observe the general trend is nearly identical when we compared the maximum and mean value graphs, but this might be due to there is not damping involved in our oscillation. For the purpose of analyzing the population change, analysis of mean value should be effective enough.
- For the analysis of the period, we could see that the period of the prey of predator is almost the same; hence we could just study one of them as they should have same period but maybe different phase.
- Above comments might be trivial, because the limiting factor of our computation efficiency is the solver for the ode. Hence it only take a bit more computational step to get extra data we might not be needed for our analysis

Conclusion

- After all the experiments, we have gained a good understanding on this local LV equation behaviour. And we should be able to characterized each parameter, and tune them to suit our system design if required. E.g. changing "a" for maximum amplitude, changing "c" for maximum period.
- Further analysis might still be needed for a better understanding.

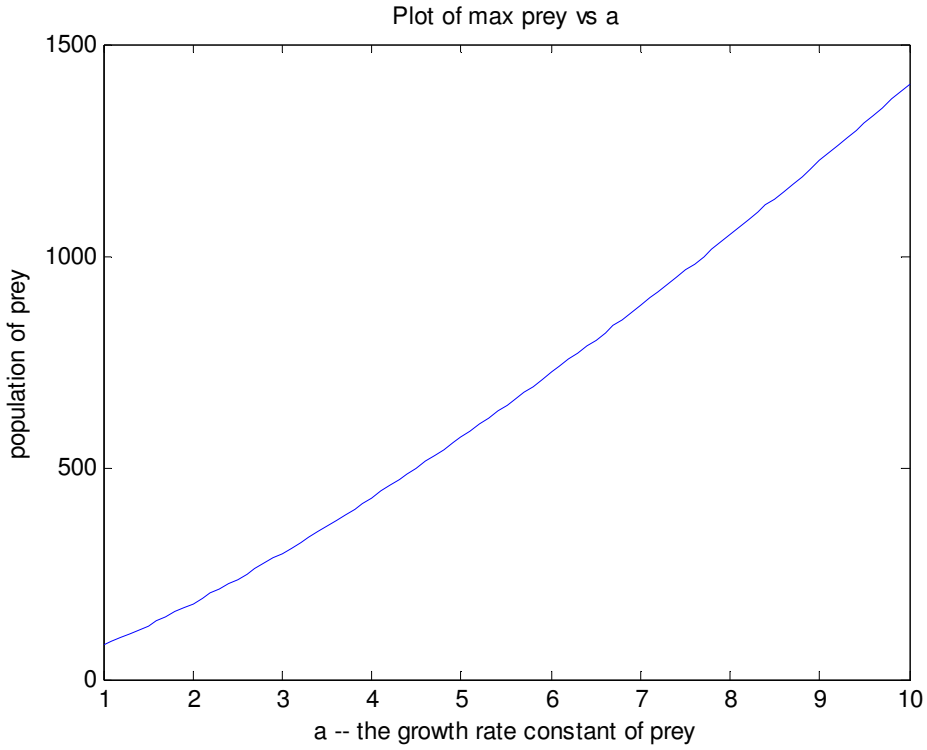
Future improvement or analysis could be made

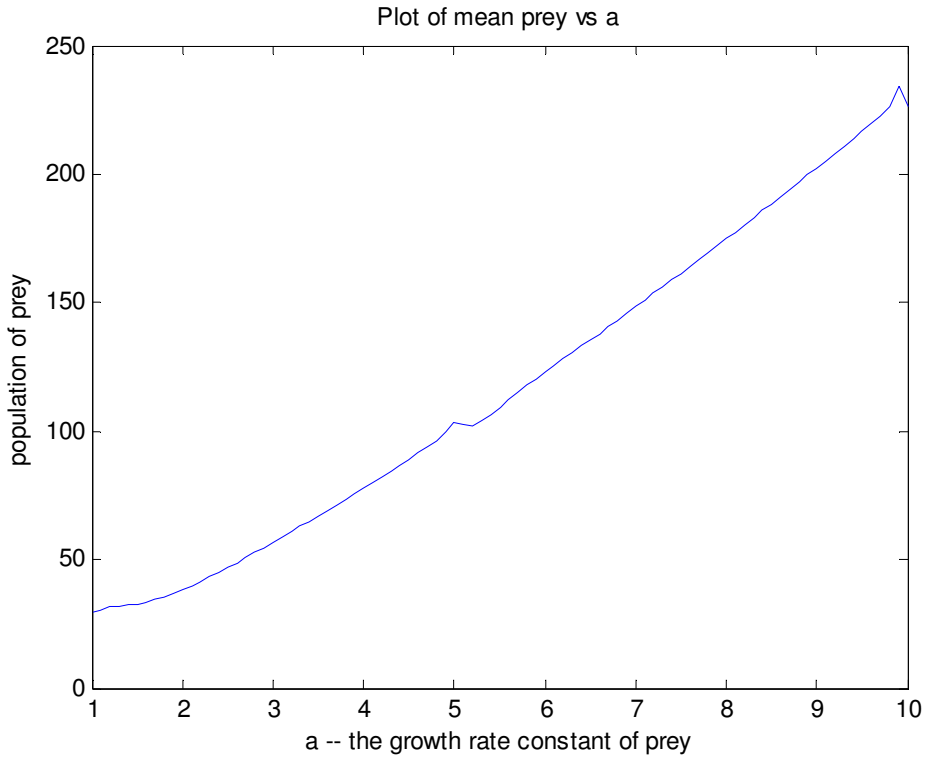
- Further analysis could be done by extreme value analysis by setting other parameters ten times more than the initial condition to check whether the graph behave correctly.
- Analysis of damping effect should be further characterized, such that change of magnitude over ten periods rather than just taking maximum and minimum point.

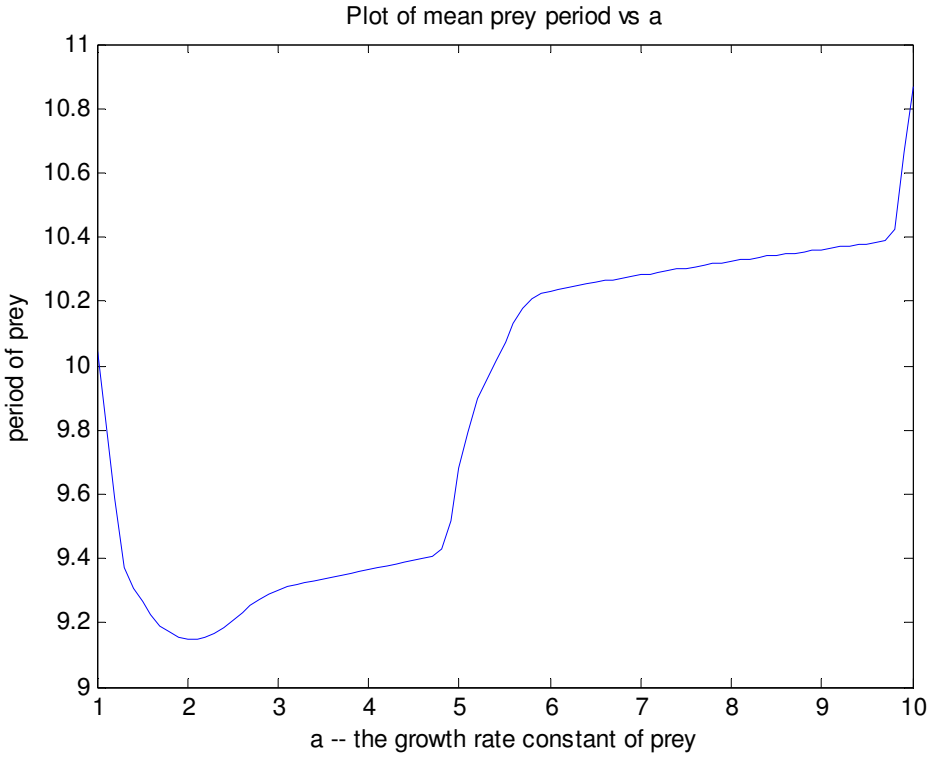
- Especially for analysis of parameter “d”, we still could observe the increased in the predator population even we increased the death rate of predator. This tells us that each parameter have its different strength. In this context, the effect of “d” is overwritten by other parameters. Further study might be needed to study for each parameter’s strength, and quantitatively obtain the threshold value on each parameter’s effect. But again this depends on our interest on the range of parameters we want to vary.
- Lastly, the effect on the initial population might be crucial for our analysis as well.

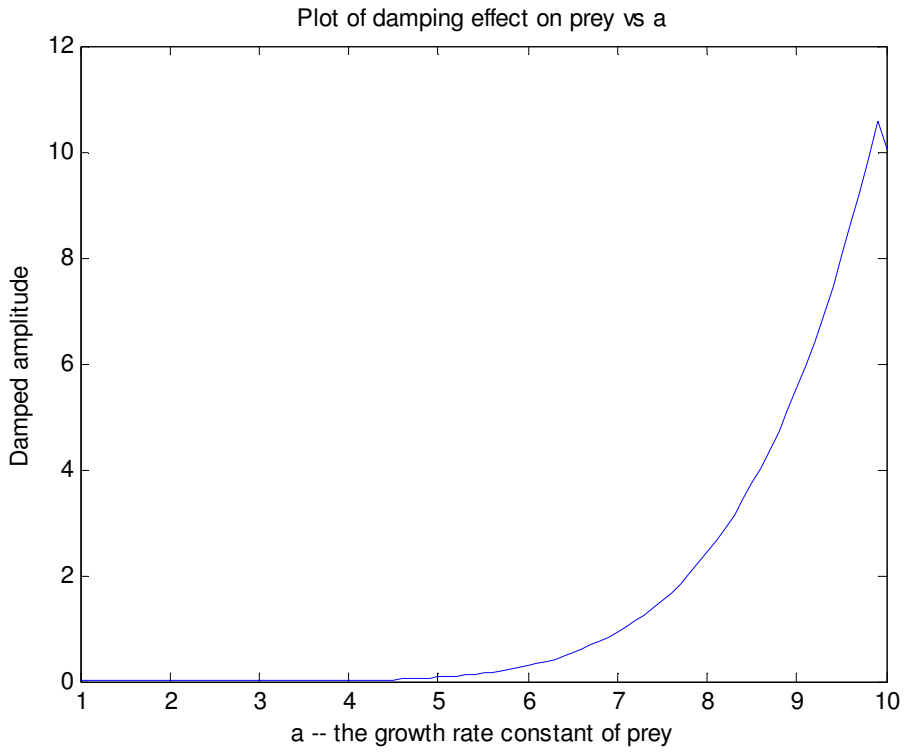
Appendix

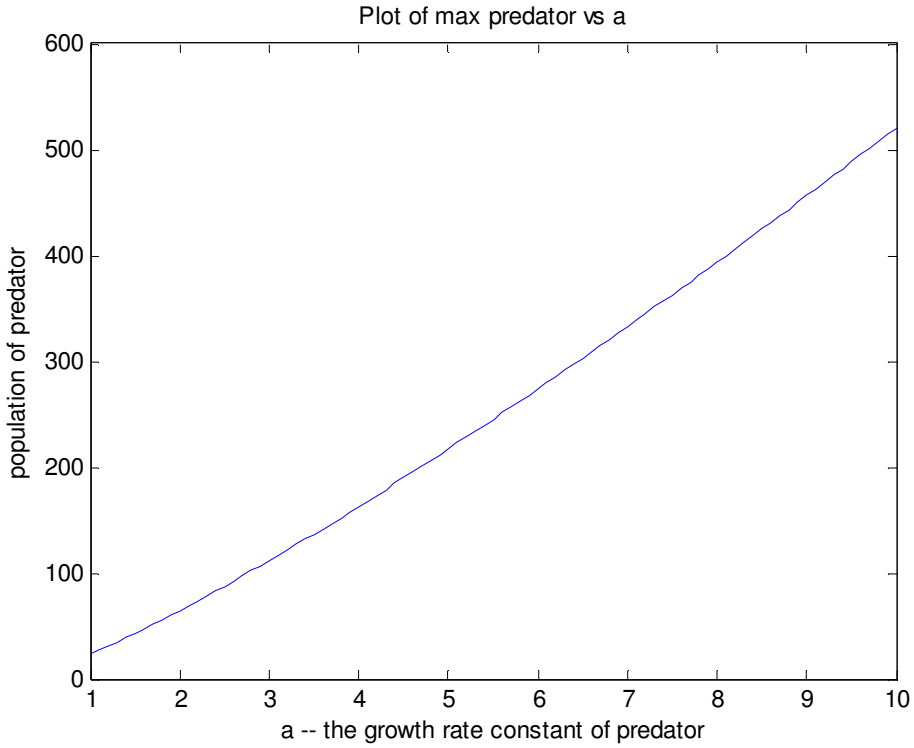
Parameter "a"

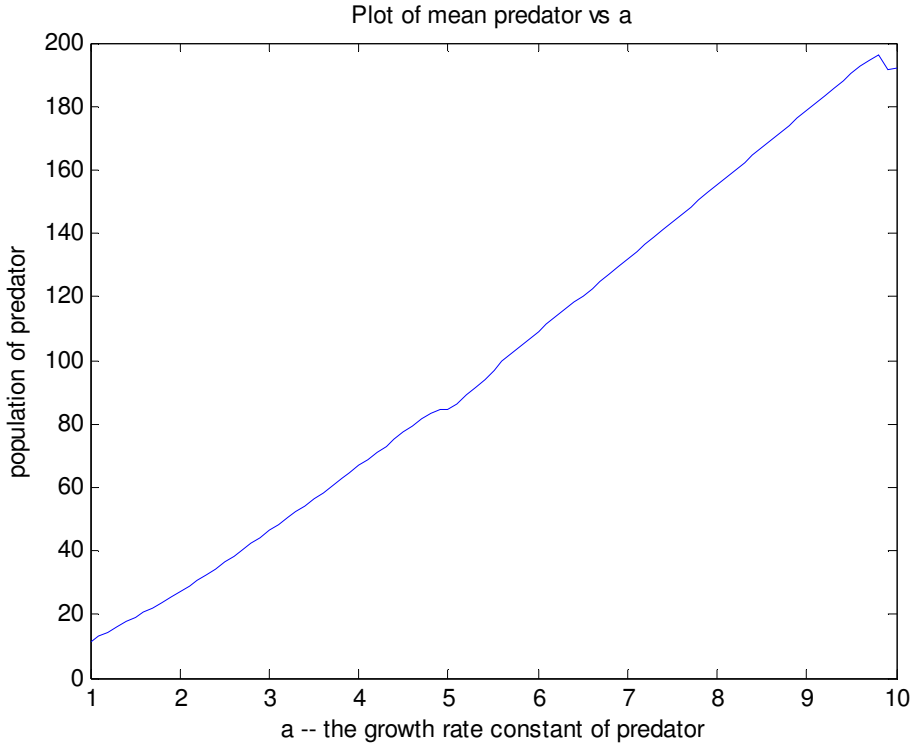


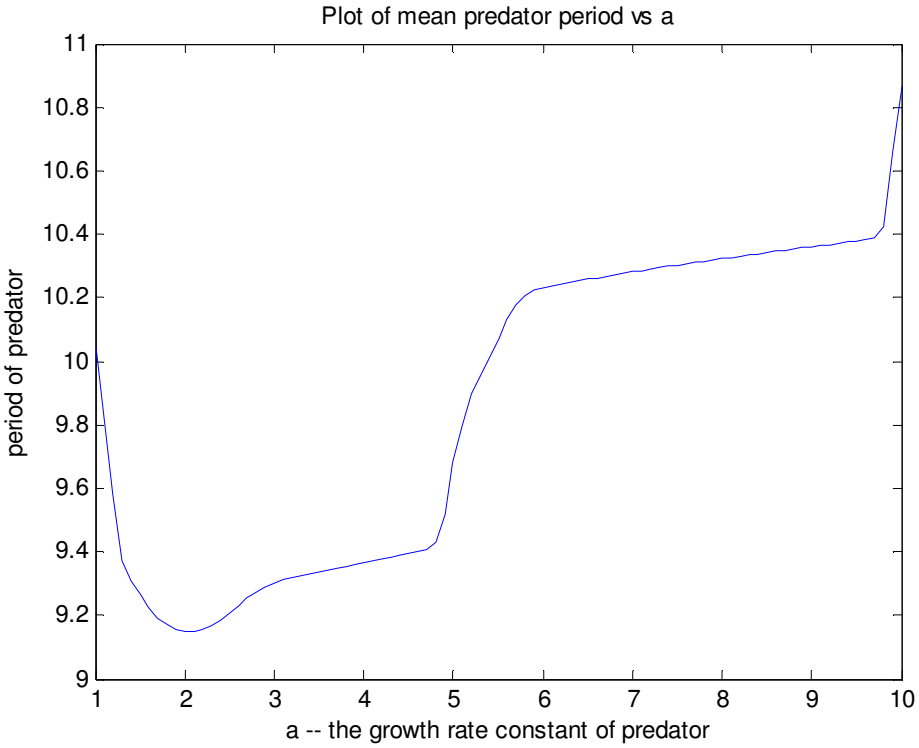


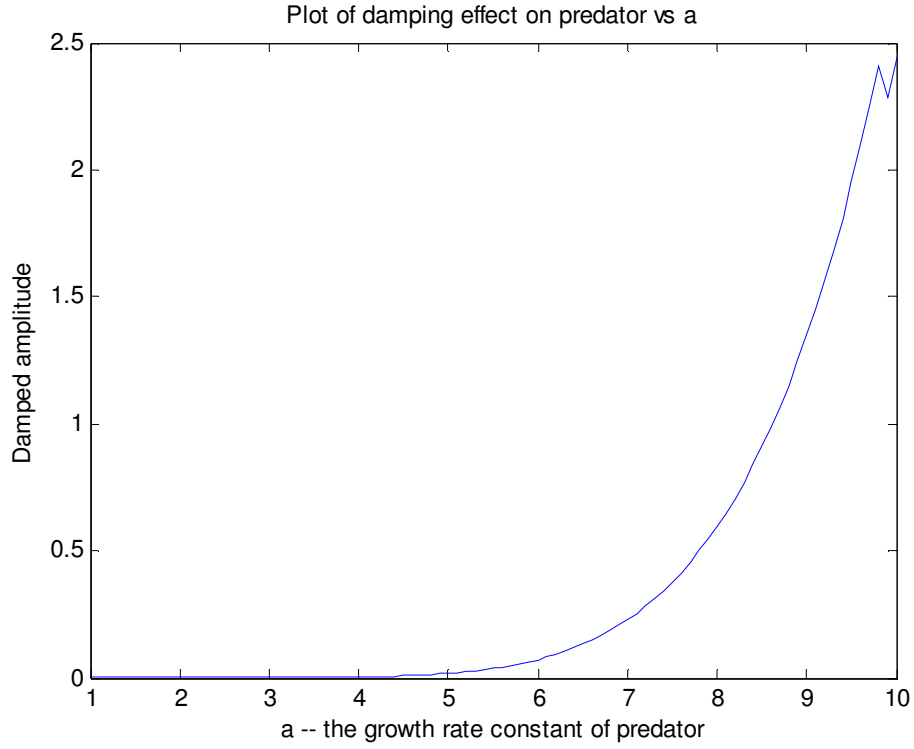

Plot of mean prey period vs a
period of prey
a -- the growth rate constant of prey
11
10.8
10.6
10.4
10.2
10
9.8
9.6
9.4
9.2
9
1
2
3
4
5
6
7
8
9
10


Plot of damping effect on prey vs a
Damped amplitude
a -- the growth rate constant of prey
12
10
8
6
4
2
0
1
2
3
4
5
6
7
8
9
10


Plot of max predator vs a
population of predator
a -- the growth rate constant of predator
0
100
200
300
400
500
600
1
2
3
4
5
6
7
8
9
10

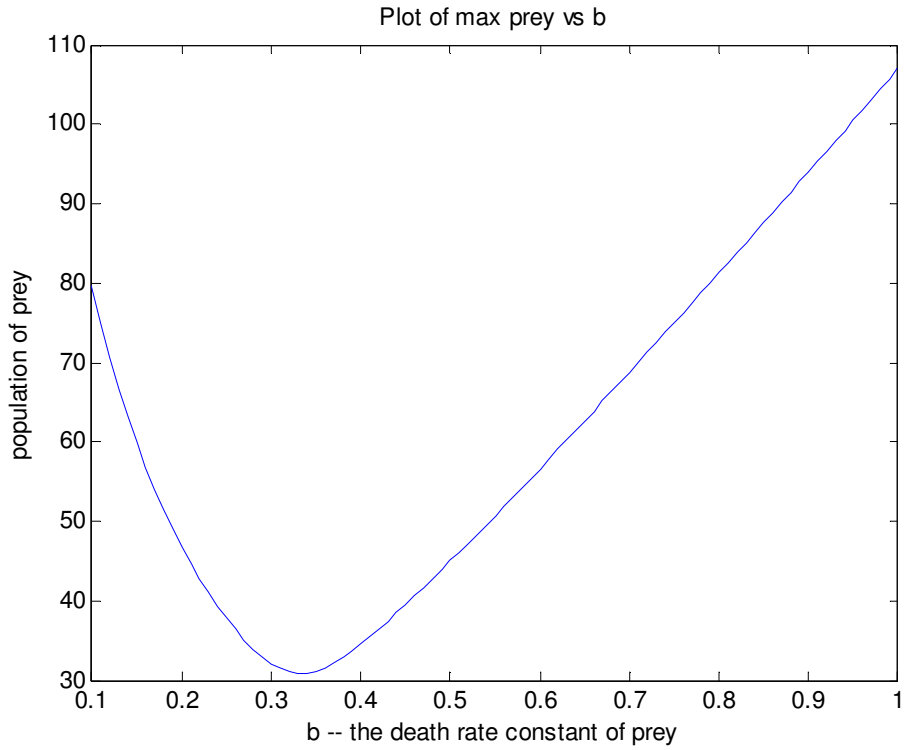

Plot of mean predator vs a
population of predator
a -- the growth rate constant of predator
0
20
40
60
80
100
120
140
160
180
200
1
2
3
4
5
6
7
8
9
10

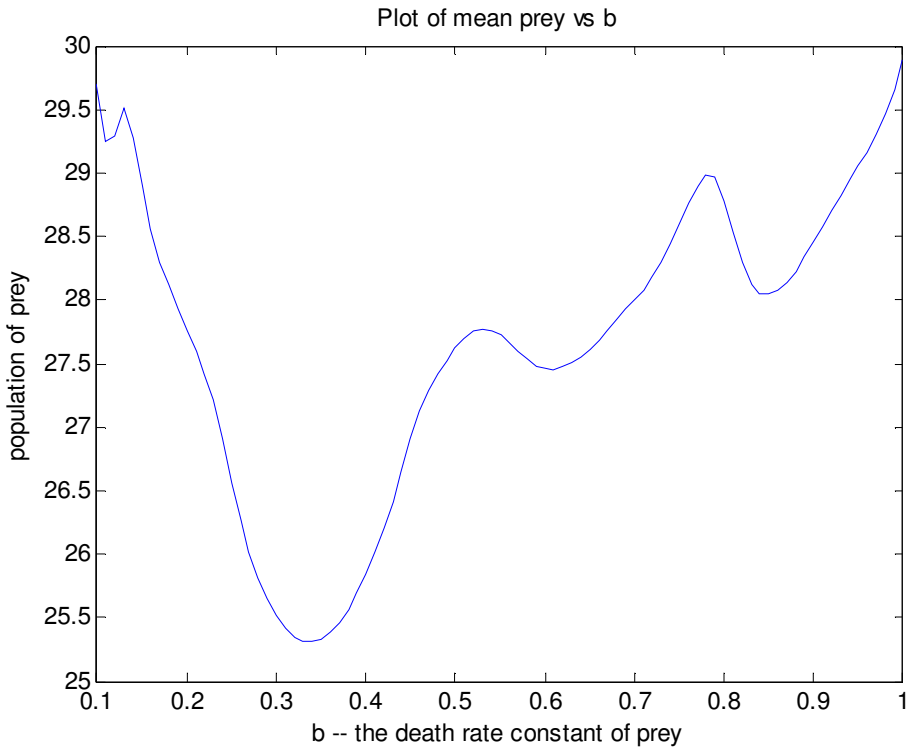


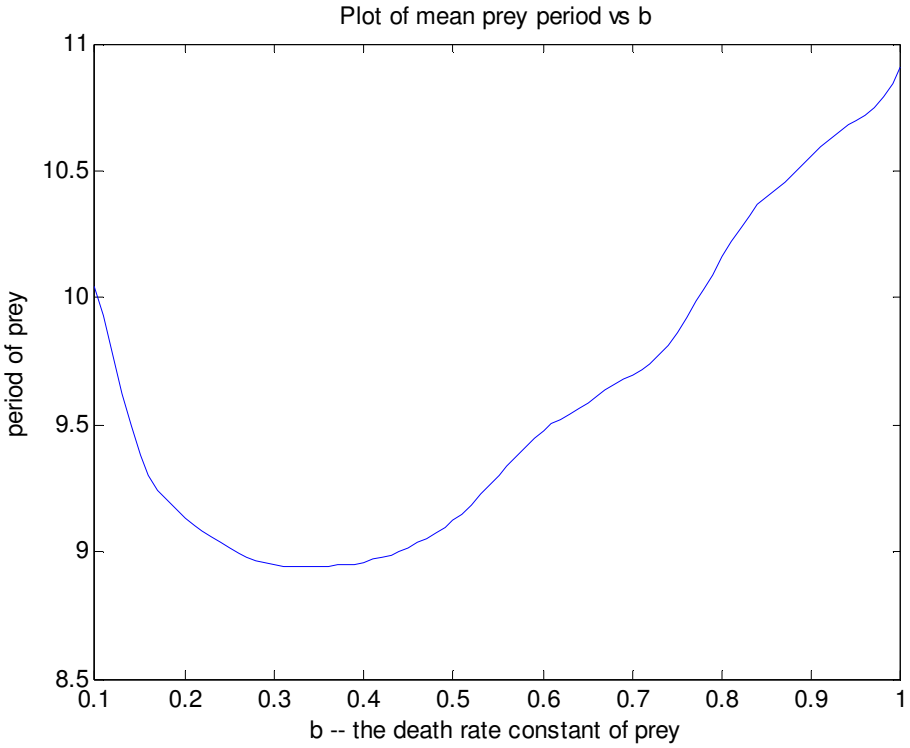


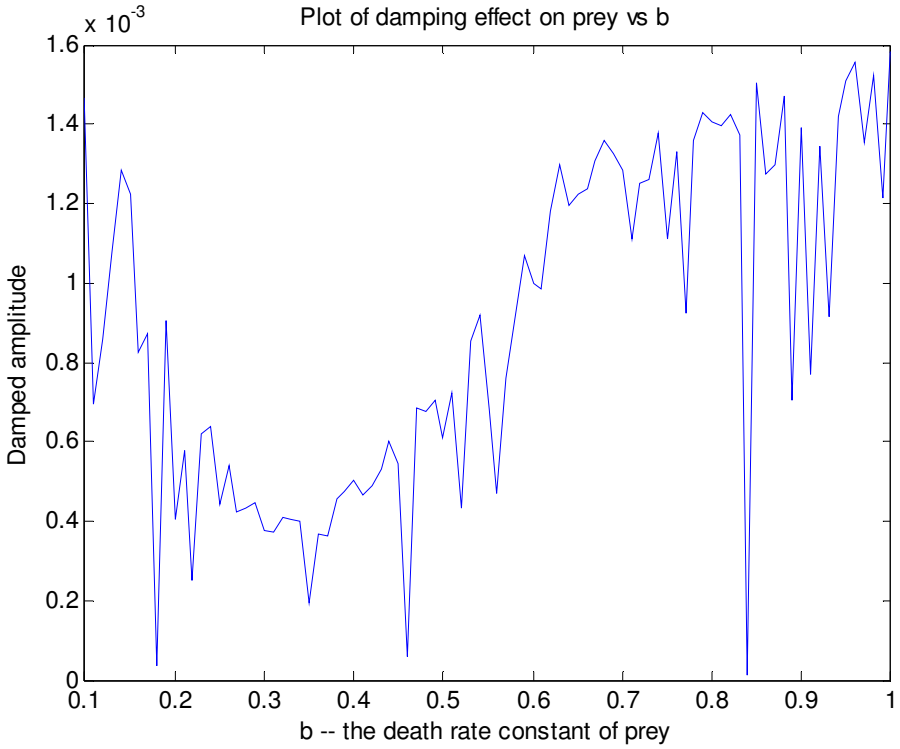
End of analysis of parameter "a"

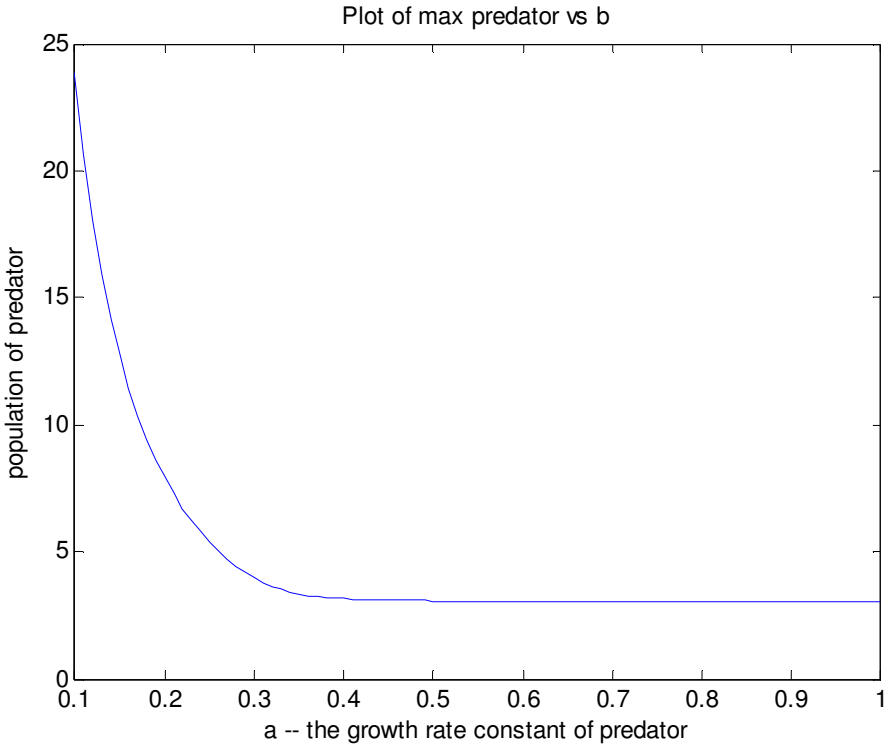
Parameter "b"

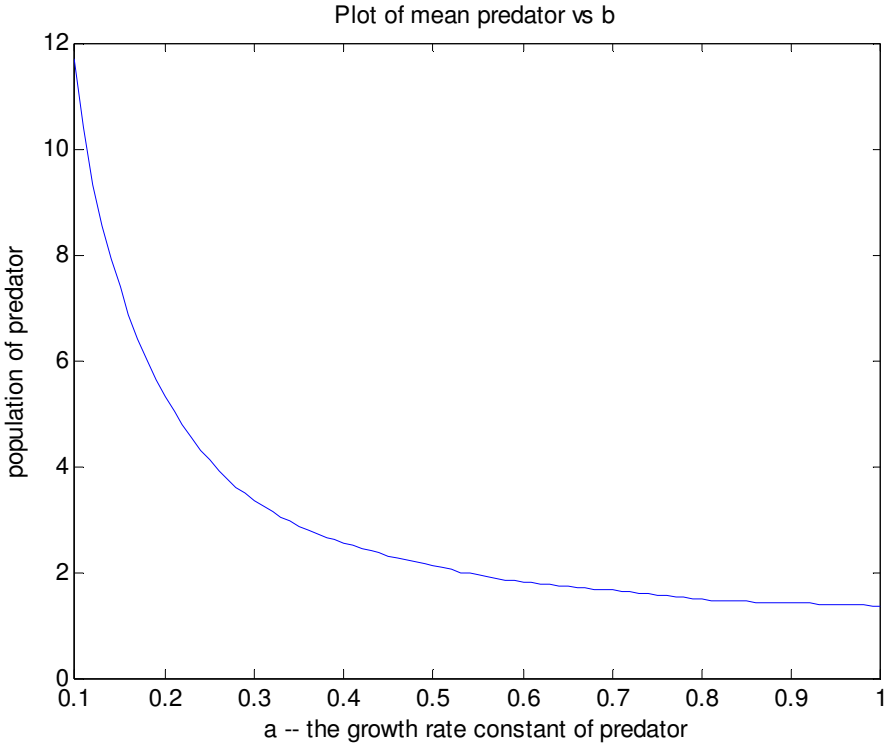

Plot of max prey vs b
population of prey
b -- the death rate constant of prey
110
100
90
80
70
60
50
40
30
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

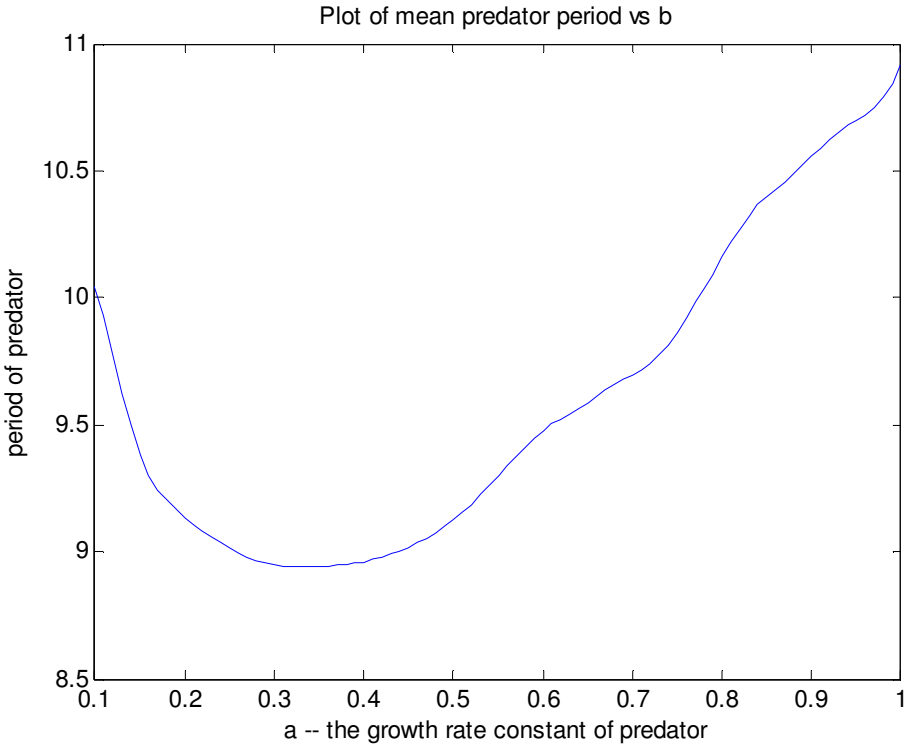

Plot of mean prey vs b
population of prey
b -- the death rate constant of prey
30
29.5
29
28.5
28
27.5
27
26.5
26
25.5
25
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

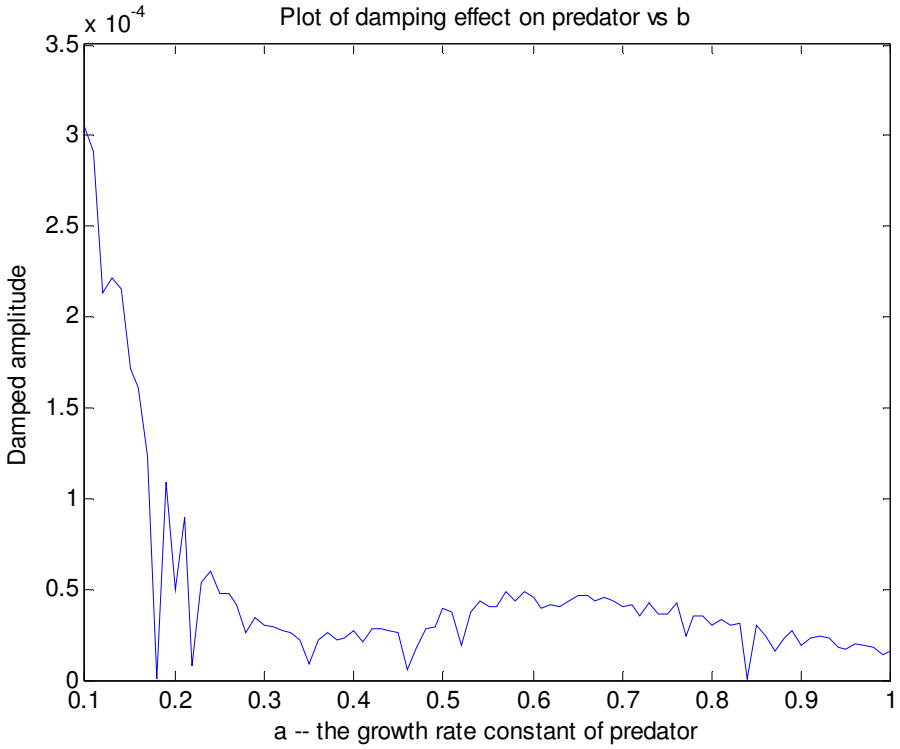

Plot of mean prey period vs b
period of prey
b -- the death rate constant of prey
11
10.5
10
9.5
9
8.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Plot of damping effect on prey vs b
x 10^-3
Damped amplitude
b -- the death rate constant of prey
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Plot of max predator vs b
population of predator
a -- the growth rate constant of predator
0
5
10
15
20
25
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

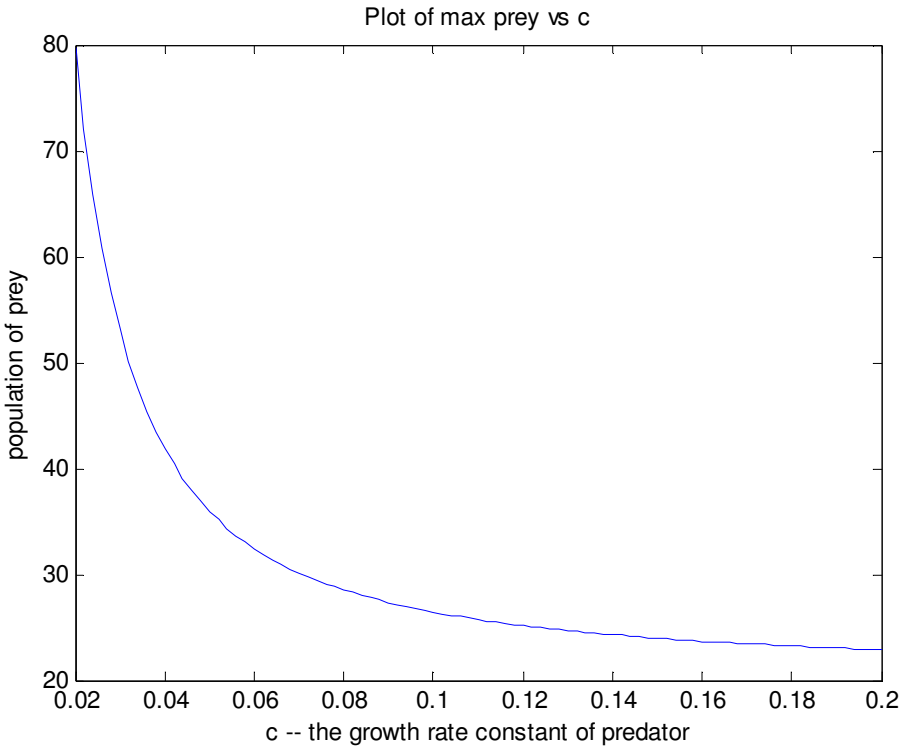

Plot of mean predator vs b
population of predator
a -- the growth rate constant of predator
0
2
4
6
8
10
12
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

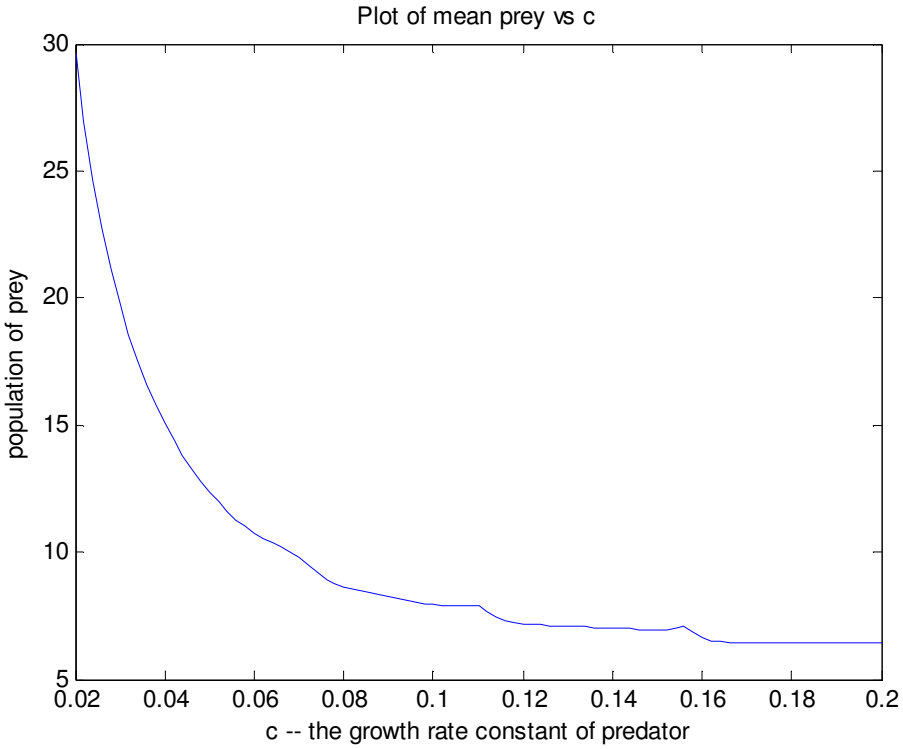


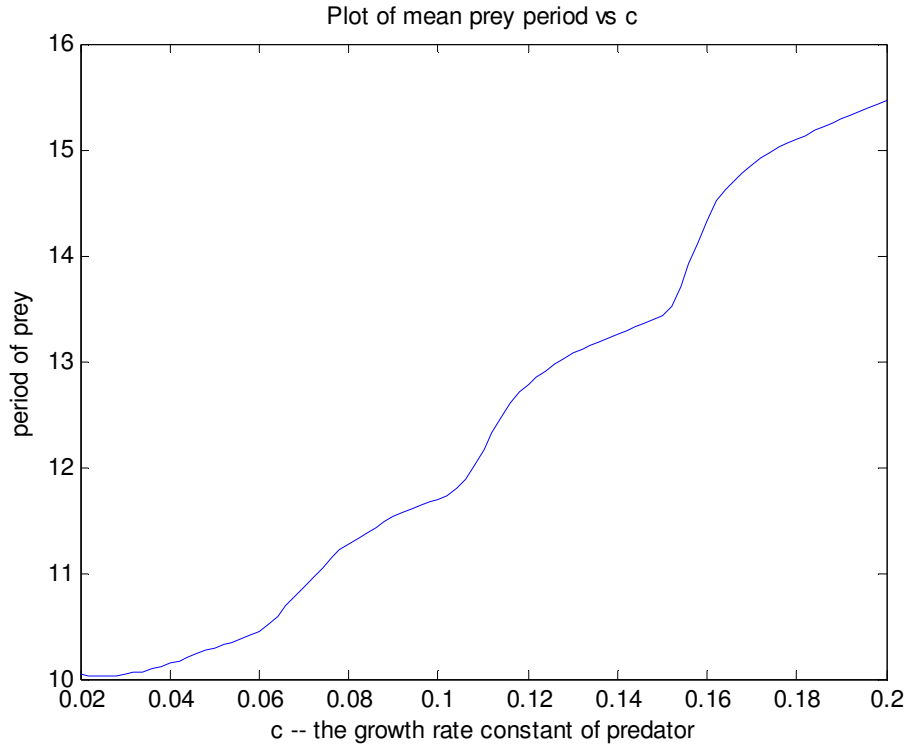


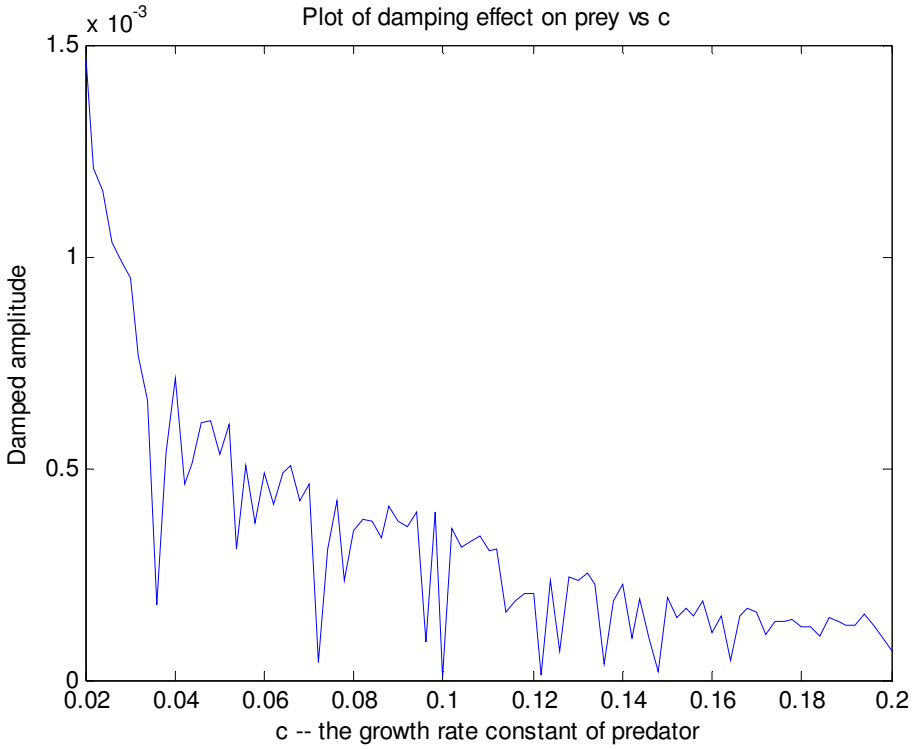
End of analysis of parameter "b"

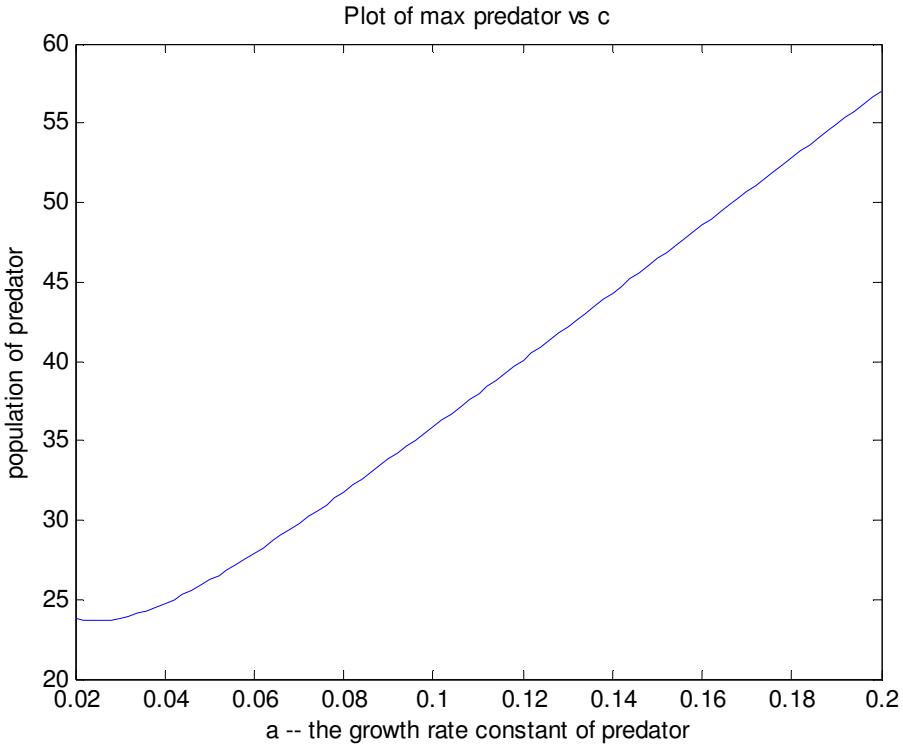
Parameter "c"

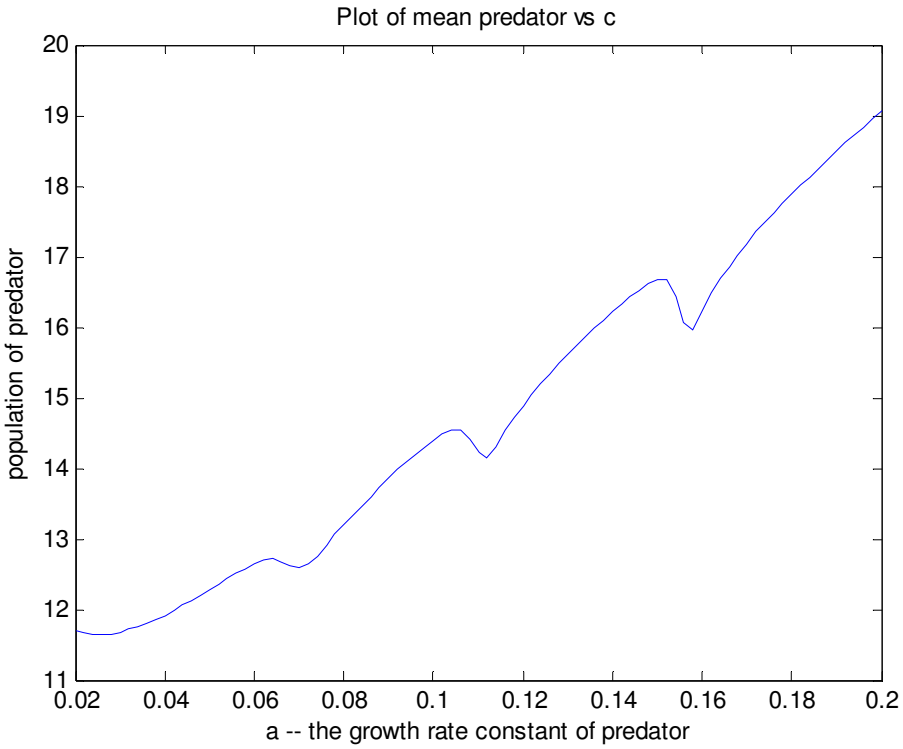

Plot of max prey vs c
population of prey
c -- the growth rate constant of predator
80
70
60
50
40
30
20
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

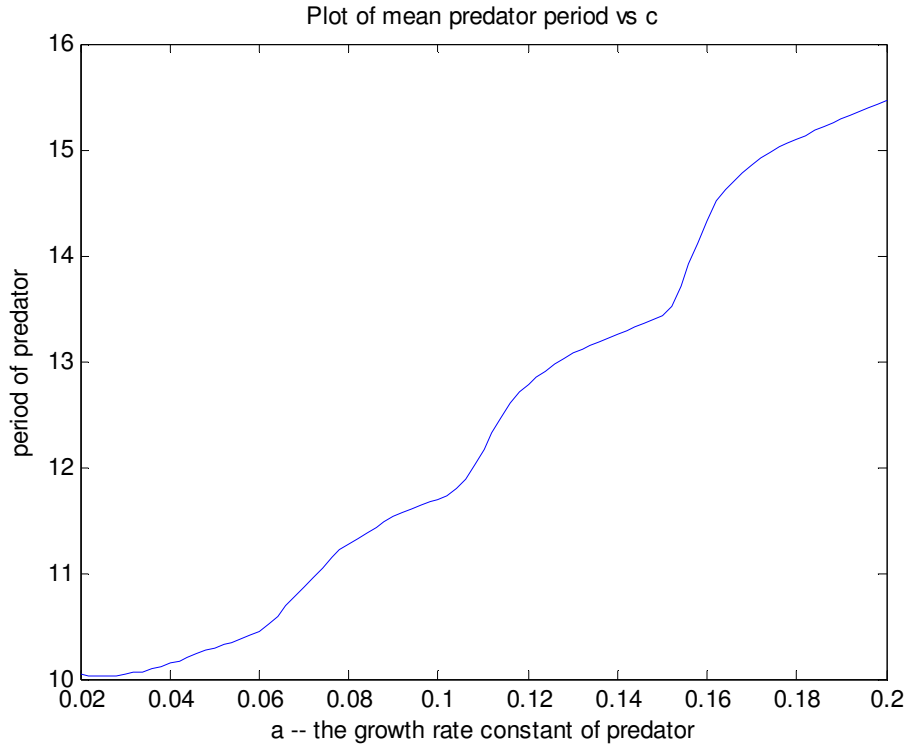

Plot of mean prey vs c
population of prey
c -- the growth rate constant of predator
30
25
20
15
10
5
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

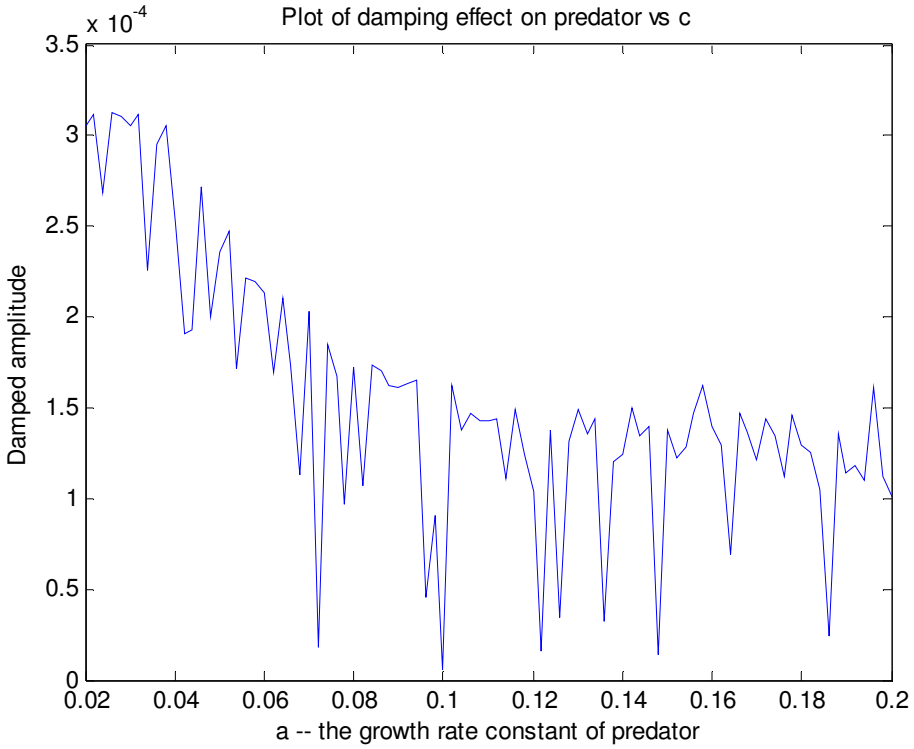

Plot of mean prey period vs c
period of prey
16
15
14
13
12
11
10
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
c -- the growth rate constant of predator


Plot of damping effect on prey vs c
x 10^-3
Damped amplitude
1.5
1
0.5
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
c -- the growth rate constant of predator


Plot of max predator vs c
population of predator
60
55
50
45
40
35
30
25
20
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
a -- the growth rate constant of predator

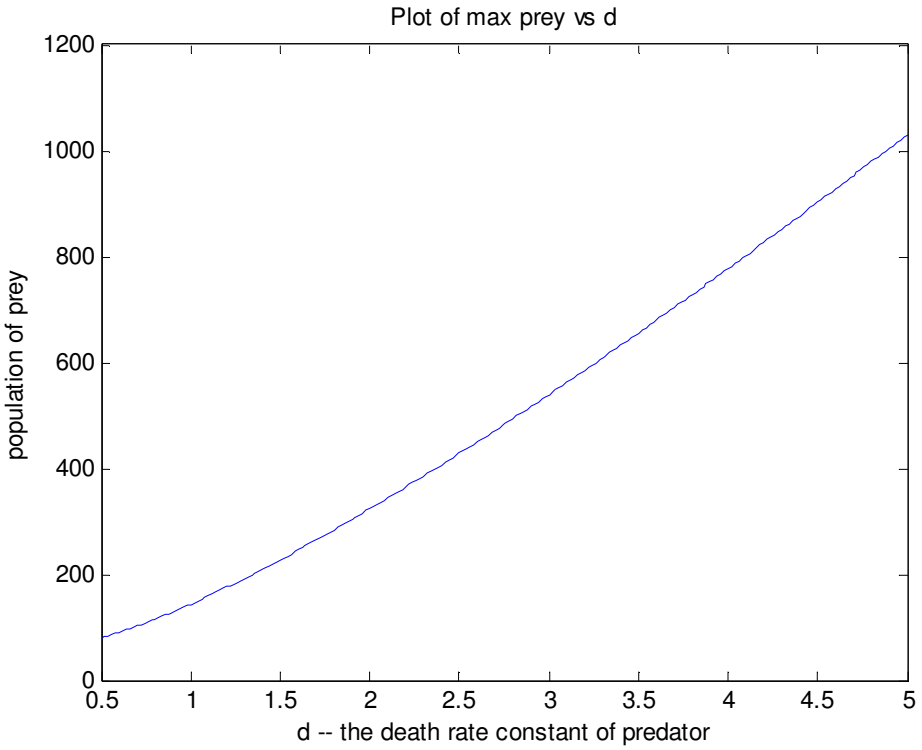

Plot of mean predator vs c
population of predator
20
19
18
17
16
15
14
13
12
11
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
a -- the growth rate constant of predator

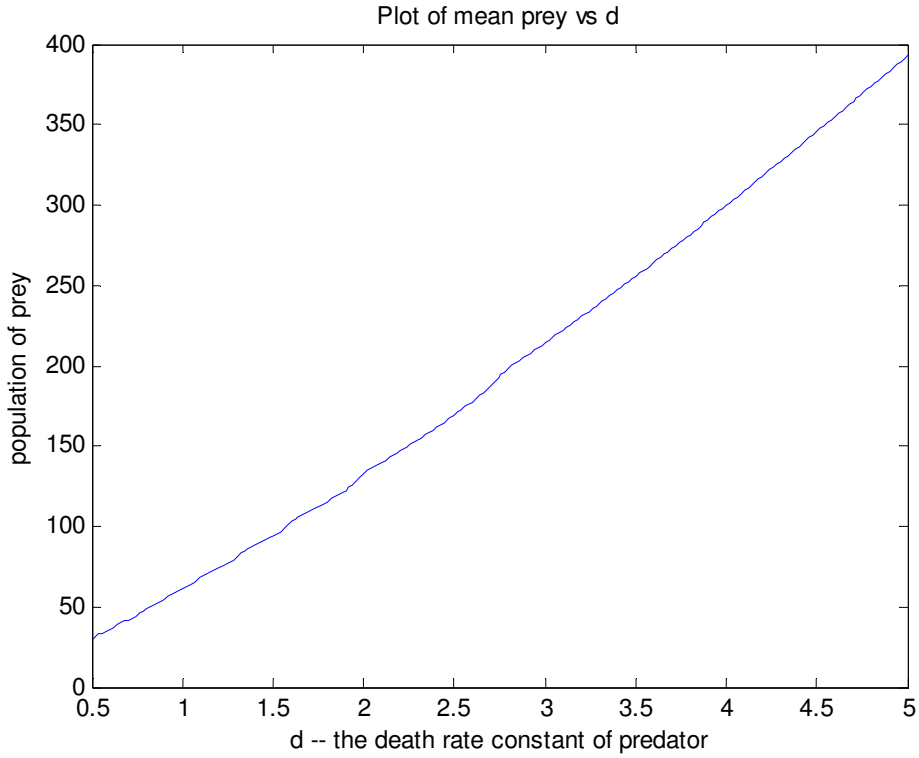


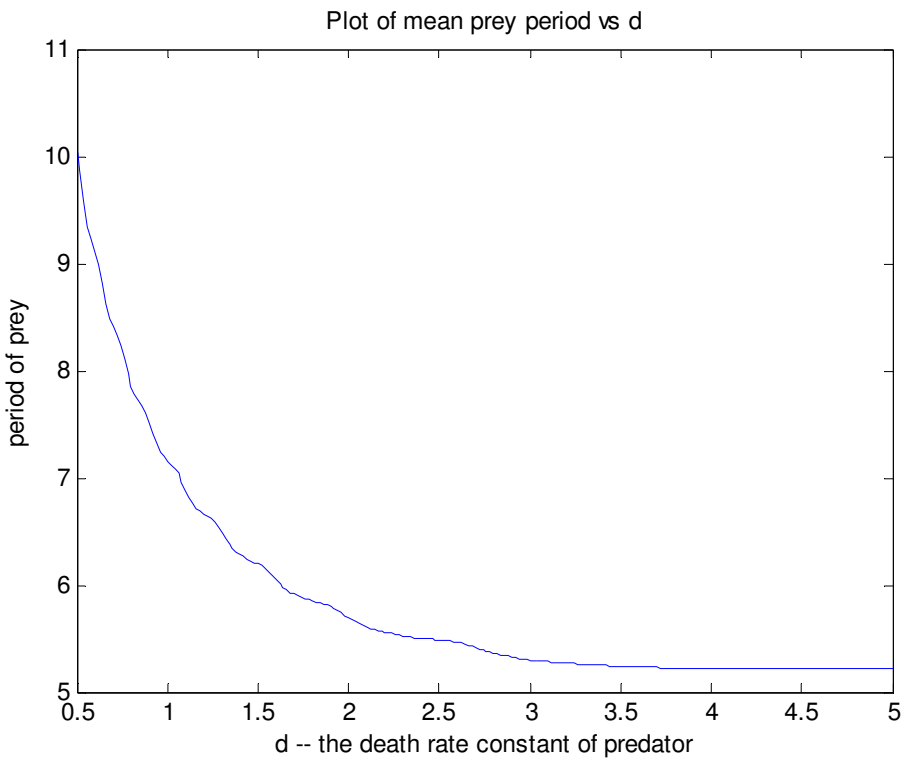


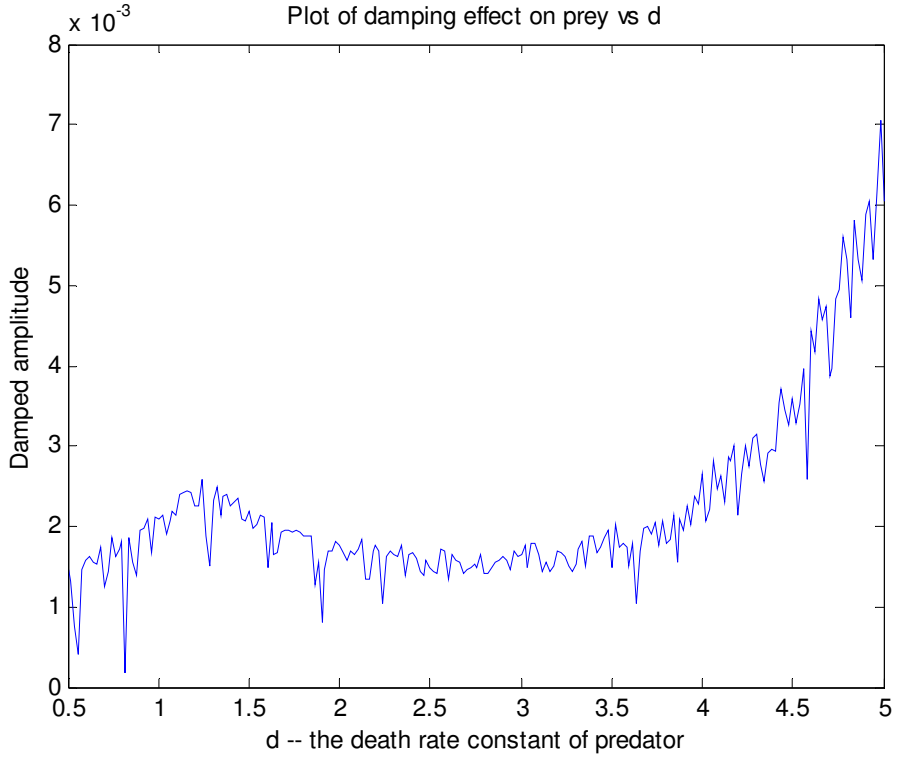
End of analysis of parameter "c"

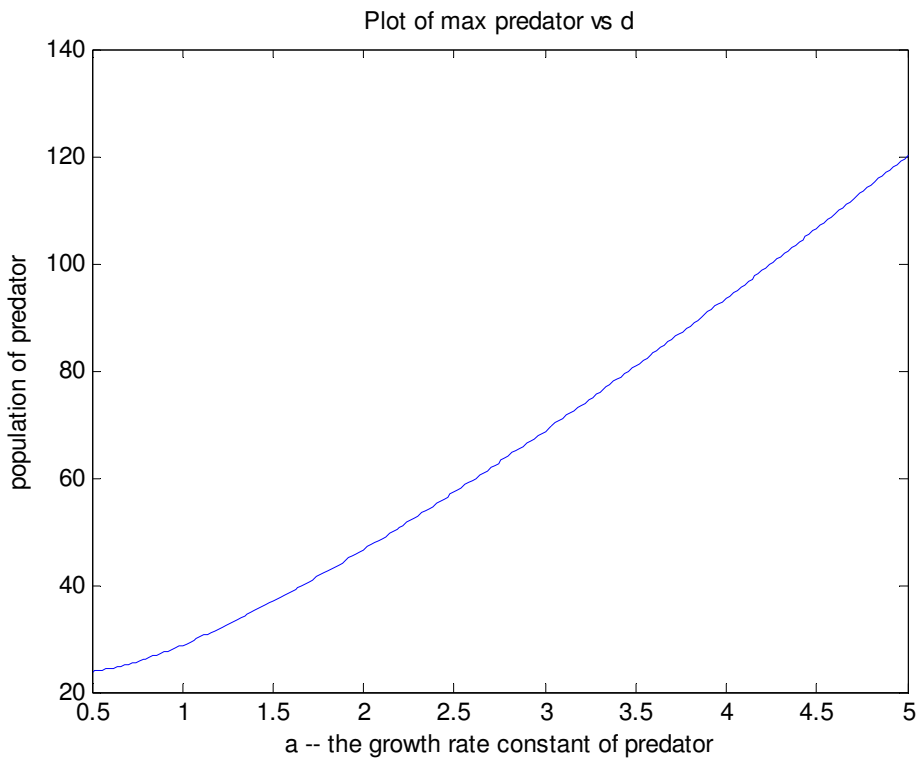
Parameter "d"

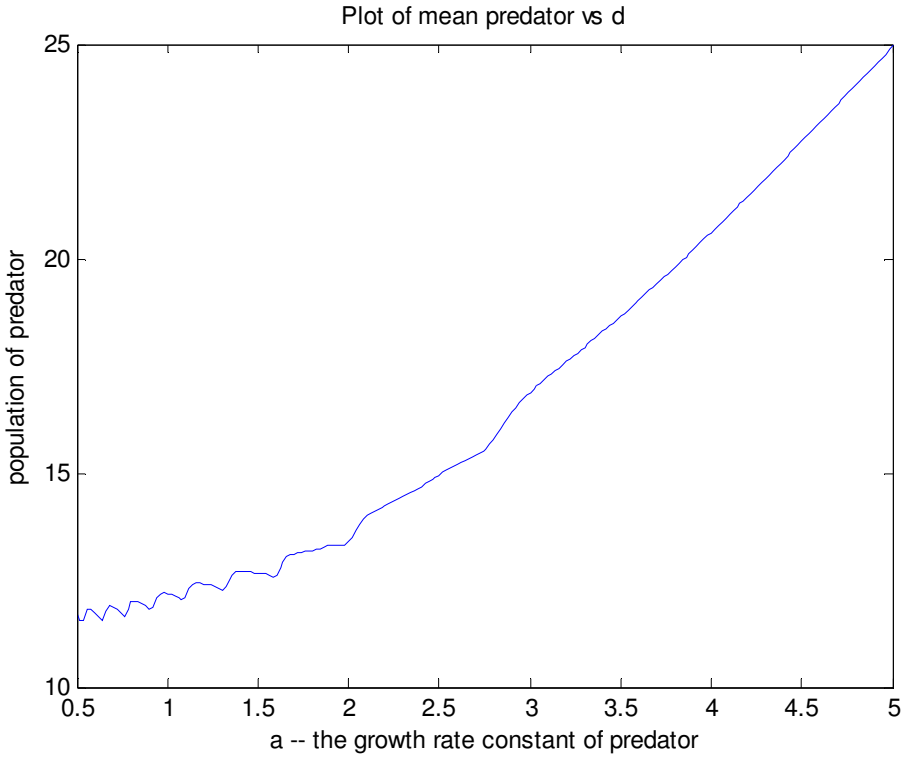

Plot of max prey vs d
population of prey
d -- the death rate constant of predator
0
200
400
600
800
1000
1200
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

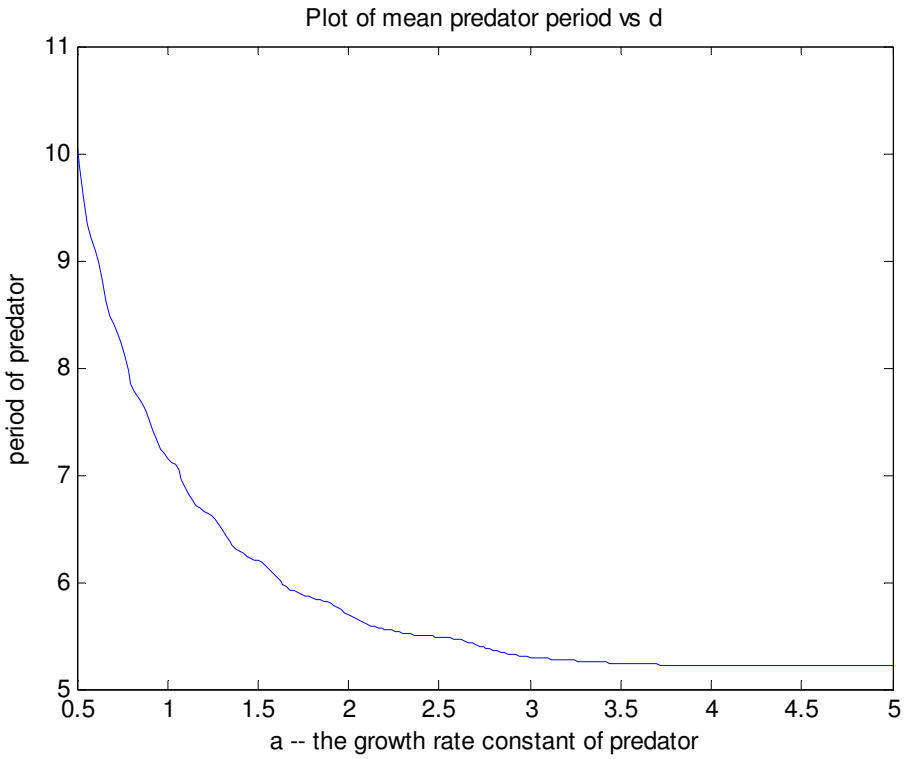

Plot of mean prey vs d
population of prey
d -- the death rate constant of predator
0
50
100
150
200
250
300
350
400
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

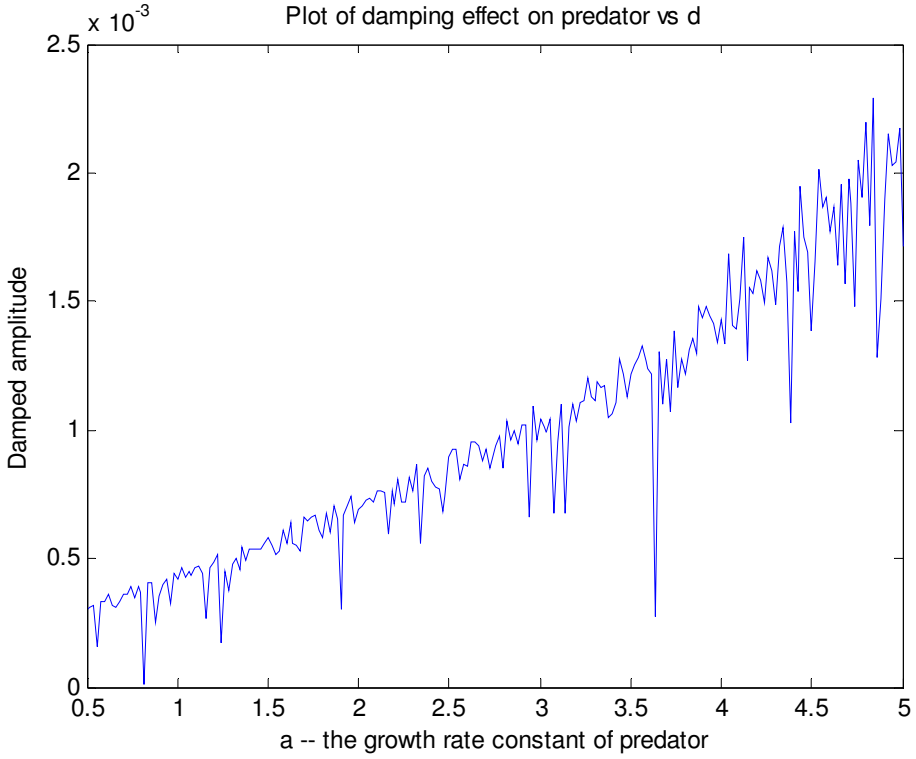

Plot of mean prey period vs d
period of prey
d -- the death rate constant of predator
11
10
9
8
7
6
5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Plot of damping effect on prey vs d
x 10^-3
Damped amplitude
d -- the death rate constant of predator
8
7
6
5
4
3
2
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Plot of max predator vs d
population of predator
140
120
100
80
60
40
20
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
a -- the growth rate constant of predator


Plot of mean predator vs d
population of predator
25
20
15
10
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
a -- the growth rate constant of predator





End of analysis of parameter "d"